

MCAT Exemplar (91027)

This is a modified version of the 2014 (Day 1) MCAT paper that has been annotated to show teachers and candidates the changes in the style of questions that can be expected in the 2015 assessment.

Note the changes to the revised front page of the assessment. These summarise some of the expectations of the candidate's work.

The candidate will not be able to achieve this standard without applying algebraic procedures in solving problems

The standard states that applying algebraic procedures involves:

- selecting and using procedures in solving problems
- demonstrating knowledge of algebraic concepts and terms
- communicating solutions using appropriate mathematical symbols.

The focus has changed from the procedures listed in explanatory note 4 (EN4) to the requirement of solving a problem as defined in explanatory note 2 (EN2).

Selecting a procedure, as listed in EN4, that when correctly applied will lead towards a solution of the problem may gain a “**u**” grade. This must be at level 6 of the curriculum.

Directing candidates to perform a procedure listed in EN4 that leads directly to the solution can only receive a grade of “**us**” for that part of the question. The candidate has not selected the procedure. In the past each of the three questions has involved several parts that ask for the demonstration of a procedure (or skill), as well as parts that require the selection and use of procedures in solving a problem.

This year at least one of the three questions will not have any directed straight procedure-based parts and the other questions a maximum of one such part. The candidate will be able to achieve the standard only if there is some evidence of having selected and used a correct procedure in solving problems, ie at least one “**u**” grade. This means a candidate must gain at least one “**u**” grade in the paper to achieve the standard – “**us**” grades alone are not sufficient.

Note that a “**us**” grade may also be awarded for a correct answer gained by either guess-and-check OR a correct-answer-only question. Only 1 “**us**” in the paper may be gained in this way and if and only if the candidate has demonstrated the use of problem solving somewhere in the paper, ie gained at least one “**u**” grade.

Teachers and candidates are also advised to read the L1 Mathematics and Statistics Assessment Specifications 2015, which include detail about the 2015 MCAT assessment.

(<http://www.nzqa.govt.nz/nqfdocs/ncea-resource/specifications/2015/level1/91028-spc-2015.pdf>)

“To meet the requirement of the standard with respect to solving problems, candidates will not be able to provide evidence by following a direction to solve factorised quadratics, factorise, expand, write or solve a linear equation, or simplify an expression involving the collection of like terms in response to being told to. One part in each question may direct the student to perform such procedures; but without further evidence at Achievement level, this will not be sufficient for the award of the standard. Utilising procedures such as factorising, in simplifying a rational function, or writing an equation from a word problem will provide evidence of solving a problem. Candidates must know that given a word problem, they will be required to write equation(s) and demonstrate consistent use of these in solving a problem. Candidates will be expected to have a basic understanding of the relationship between a quadratic function and the associated graph.”

2014

Level 1 Mathematics and Statistics

91027 Apply algebraic procedures in solving problems

Credits: Four

You should attempt ALL parts of ALL questions in this booklet.

Calculators may NOT be used.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. Guess and check methods do not demonstrate relational thinking. Guess and check methods will limit grades to Achievement.

Answers must be given in their simplest algebraic form.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

<i>For Assessor's use only</i>	Achievement Criteria		
Achievement	Achievement with Merit	Achievement with Excellence	
Apply algebraic procedures in solving problems. <input type="checkbox"/>	Apply algebraic procedures involving relational thinking, in solving problems. <input type="checkbox"/>	Apply algebraic procedures involving extended abstract thinking, in solving problems. <input type="checkbox"/>	
Overall Level of Performance			<input type="checkbox"/>

You are advised to spend 60 minutes answering the questions in this booklet.

QUESTION ONE

Note: For this question, only one of parts (a), (b) or (c) can count as a “us” (achievement level skills grade.)

These do not satisfy the requirement of the candidate selecting a procedure.

(a) Simplify $5m^2 n \times m^3 n^2$

(b) Solve $4a^3 = 32$

a =

(c) Solve $8 = \frac{5x - 4}{2}$

x

(d) Factorise $3a^2b + a^3 b^2 - 5a^2 b$

Accept for a “u” grade if it is correctly simplified but not factorised– they have chosen this as a correct procedure on the way to a correct answer.

If correctly factorised but not simplified it counts as “us”,
They were directed to factorise,

Simplified and factorised, it gains “r”,

Note the original instruction “Give your answer in the simplest form” has been deleted as this directs the procedure.

- (e) Mark had worked twice as many hours as James.
If James had worked another 48 hours he would have worked twice as long as Mark.

The original line "Write an equation, and use this to find how many more hours Mark worked than James" must be replaced with:

How many more hours had Mark worked than James?

Then it is fine, but the student must show how they have used algebra, in solving the problem.

- (f) Solve $\frac{4x - 6}{3} \geq 2x + 1$

This is fine because it needs several procedures that are not specified.

- (g) Simplify

$$\frac{r^2 - 1}{r^2 + r}$$

While this question tells the candidate to simplify (One of the selection of procedures listed), this is fine as it involves factorising as well as simplifying which is not specified.

QUESTION TWO

- (a) Factorise $x^2 - 3x - 40$ This is not curriculum level 6 - not acceptable question
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- (b) Sam is paid \$25 to make 5 deliveries, for a chemist shop after school.
He also receives \$2 for each delivery he makes.

Note that neither parts (i) or (ii) below, can be asked, as these are too directed – not problem solving.

The question could be modified to:

“How many deliveries must he have made if Sam earned \$47?”

Candidates must know to show the use of algebra they cannot be directed to do this.

If guess-and-check is used they could gain “us”.

- i) Give the formula for the wages that he receives
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-
-
-

- ii) Make d the subject of the formula you wrote in part (i).
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-
-
-

- (c) Emma tells her friend that her height is at least as much as her younger sister’s plus a quarter as much again.

Part c) (i) was too directive as it told candidates to write an inequation (“Write an inequation to express Emma’s height E in terms of the height of her sister, S .”)

A possible modified question:

Her sister’s height is 96 cm.

- (i) Find Emma’s height.

Note: If they do not write an equation and demonstrate the use of algebra the candidate can only gain evidence for “us” from either guess-and-check or correct-answer-only.

(d) An n -sided polygon has $D = \frac{n}{2} (n - 3)$ diagonals.
How many sides has the polygon if there are 20 diagonals?

This question is fine as is.

QUESTION THREE

Note: For Question Three, only one of parts (a) or (b) can count as a “us” grade. However one question in the 2015 paper will not contain any “us” questions.

(a) Simplify $\frac{3x}{2} + \frac{2x}{5}$

(b) Simplify $(2x^2)^3$

(c) Solve $3 \times 2^{a-1} = 96$

This question is OK as the candidate must simplify the equation before forming a relationship for the powers of 2.

(d) Peter is investigating sequences of numbers. One of the sequences is listed below:

Number, N	Sequence, U	Prime Number?
1	23	YES
2	25	NO=5x5
3	29	YES
4	35	NO=5x7
5	43	YES

The formula for this sequence is $U = n^2 - n + 23$.

- i) What is the value of U for the 12th number in the sequence? **This question is not appropriate, as not at curriculum level 6.**

Next two question parts are fine:

Some of the numbers are prime numbers.
(A prime number is one that can only be divided by 1 and itself. 1 is not a prime number.)

For the sequence of numbers where

$$U = n^2 - n + a$$

Show that when $n = a$ then U will never be a prime number.

Assume $n > 1$.

- ii) If $T = n^2 - n + 5$ and $R = (5n - 4)(n + 1) - 2n(2n+3) + 4(n + 1) - 3$

Write an equation for R in terms of T.

- iii) **Have deleted "Using the formula..." from the original question.**
If candidates do not use the formula then they cannot gain more than "us".

$U = n^2 - n + 1$ find the value of n when $U = 91$

iv) **This is fine:**
Explain why
$$U = n^2 - n - 6$$

will never give a positive prime number value for U.

91027 MCAT PAPER
ADAPTED FROM 2014 DAY 1 PAPER
TO MATCH THE REQUIREMENTS FOR 2015

Credits: Four

You should answer ALL parts of ALL questions in this booklet.

You should show ALL working.

CALCULATORS MAY NOT BE USED.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper.

Guess and check and correct answer only methods do not demonstrate relational thinking and will limit the grade for that part of the question to a maximum of an achieved grade.

Guess and check and correct answer only may only be used a maximum of one time in the paper and will not be used as evidence of solving a problem.

A candidate cannot gain achievement in this standard without solving at least one problem.

Answers must be given in their simplest algebraic form.

Where a question is given in words you will be expected to write an equation.

YOU MUST HAND THIS BOOKLET TO YOUR SUPERVISOR AT THE END OF THE ALLOTTED TIME.

<i>For Assessor's use only</i>	Achievement Criteria	
Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic procedures in solving problems. <input style="float: right;" type="checkbox"/>	Apply algebraic procedures using relational thinking, in solving problems. <input style="float: right;" type="checkbox"/>	Apply algebraic procedures using extended abstract thinking, in solving problems. <input style="float: right;" type="checkbox"/>
Overall Level of Performance		<input type="checkbox"/>

You are advised to spend 60 minutes answering the questions in this booklet.

QUESTION ONE

a) Solve $8 = \frac{5x - 4}{2}$

x

b) Factorise $3a^2b + a^3b^2 - 5a^2b$

c) Mark had worked twice as many hours as James.
If James had worked another 48 hours he would have worked twice as long as Mark.

How many more hours had Mark worked than James?

d) Solve $\frac{4x - 6}{3} \geq 2x + 1$

x

e) Simplify

$$\frac{r^2 - 1}{r^2 + r}$$

QUESTION TWO

- a) Between what values of x will y be positive if
 $y = (3 - 2x)(x + 1)$

- b) Sam is paid \$25 to make 5 deliveries d , from a chemist shop after school.
He also receives \$2 for each additional delivery he makes.
How many deliveries must he have made if he earned \$47?

- c) Emma tells her friend that her height is at least as much as her younger sister's plus a quarter as much again.
Her sister's height is 96 cm.
Compare Emma's height with her sister's.

- d) An n -sided polygon has $D = \frac{n}{2}(n - 3)$ diagonals.
How many sides has the polygon if there are 20 diagonals?

QUESTION THREE

a) Factorise $2x^2 + x - 3$

b) Peter is investigating sequences of numbers.
One of the sequences is listed below:

Number, N	Sequence, U	Prime Number?
1	23	YES
2	25	NO=5x5
3	29	YES
4	35	NO=5x7
5	43	YES

The formula for this sequence is $U = n^2 - n + 23$.

Some of the numbers are prime numbers.
(A prime number is one that can only be divided by 1 and itself. 1 is not a prime number.)

i) For the sequence of numbers where
 $U = n^2 - n + a$

Show that when $n = a$ then U will never be a prime number.

Assume $n > 1$.

ii) If $T = n^2 - n + 5$ and $R = (5n - 4)(n + 1) - 2n(2n+3) + 4(n + 1) - 3$

Write an equation for R in terms of T.

iii) $U = n^2 - n + 1$ find the value of n when $U = 91$

iv) Explain why
 $U = n^2 - n - 6$

will never give a positive prime number value for U.

Mathematics and Statistics 1.2: Apply algebraic methods in solving problems

Achievement	Merit	Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Evidence Statement

Question One	Evidence	Achievement Apply algebraic methods in solving problems by ...	Merit Apply algebraic methods, using relational thinking, in solving problems by ...	Excellence Apply algebraic methods, using extended abstract thinking, in solving problems by ...
(a)	$16 = 5x - 4$ $x = 4$	Equation solved us		
(b)	$a^3 b^2 - 2 a^2 b$ $= a^2 b (ab - 2)$	Simplified or factorised	Simplified and factorised	
(c)	$M = 2J$ $J + 48 = 2M$ $= 4J$ $3J = 48$ $J = 16$ $M = 32$ Mark worked 16 hours more than James.	Equation given relating M and J	Equation solved	Conclusion correct
(d)	$4x - 6 \geq 6x + 3$ $-9 \geq 2x$ $x \leq -4.5$	Expression rearranged and expanded.	Inequation solved	
(e)	$\frac{(r+1)(r-1)}{r(r+1)}$ $= \frac{r-1}{r}$ OR $1 - 1/r$	Numerator or denominator factorised	Numerator and denominator factorised	Expression simplified
		1u or us gains 1A 2 or more u or us gains 2A	1r gains 1M 2 or more r gains 2M	1t gains 1E 2 or more t gains 2E

Question Two	Evidence	Achievement Apply algebraic methods in solving problems by ...	Merit Apply algebraic methods, using relational thinking, in solving problems by ...	Excellence Apply algebraic methods, using extended abstract thinking, in solving problems by ...
(a)	$-1 \leq X \leq 3/2$	Correct interval with or without the equal signs		
(b)	$P = 25 + 2(d - 5)$ $= 15 + 2d$ $= 47$ Number of deliveries = $6 + 5$ $= 11$ $47 = 25 + 2d$ $d = 6$ If 5 added	Correct expression Incorrect equation consistently solved	Correctly solved Correctly solved	
(c)	$E \geq S + 0.25S$ $\geq 1.25 S$ $\geq 96 + \frac{96}{4}$ $\geq 96 + 24$ ≥ 120 Emma is at least 24 cm taller than her sister Accept Emma is at least 110 cm	1 correct algebraic statement	Progress towards solution	Correct solution
(d)	$40 = n^2 - 3n$ $n^2 - 3n - 40 = 0$ $(n + 5)(n - 8) = 0$ Number of diagonals must be +ve therefore 8 diagonals.	Equation rearranged	Equation correctly solved	Correct equation
		1u or us gains 1A 2 or more u or us gains 2A	1r gains 1M 2 or more r gains 2M	1t gains 1E 2 or more t gains 2E

Question Three	Evidence	Achievement Apply algebraic methods in solving problems by ...	Merit Apply algebraic methods, using relational thinking, in solving problems by ...	Excellence Apply algebraic methods, using extended abstract thinking, in solving problems by ...
(a)	$(2x + 3)(x - 1)$	Correctly factorised us		
(b)i)	$U = n^2 - n + n$ $= n^2$ n^2 is always the product of a number multiplied by itself which is not a prime number.	Correctly simplified	Clearly explained	
ii)	$R = 5n^2 + n - 4$ $- 4n^2 - 6n + 4n$ $+ 4 - 3$ $= n^2 - n - 3$ $R = T - 8$	Correct expansion of 1 set of brackets	Simplified	Correctly equated
iii)	$91 = n^2 - n + 1$ $n^2 - n - 90 = 0$ $(n + 9)(n - 10) = 0$ $n = -9$ or 10 $n = 10$	Correct rearrangement	Forming and factorising	Correct solution
iv)	$U = (n - 3)(n + 2)$ If $n > 3$ both factors are greater than 1 Hence U is the product of 2 numbers and so it is not a prime number.	Factorised		Conclusion stated
		1u or us gains 1A 2 or more u or us gains 2A	1r gains 1M 2 or more r gains 2M	1t gains 1E 2 or more t gains 2E

OVERALL GRADE.

Achievement 3 or more A ie $A + A + A$ or $2A + 1A$

Provided that at least 1 A contains a straight u grade (not us).

Only 1 us grade from guess and check or correct answer only can be counted as contributing to an Achievement grade

Merit – at least 2M

Excellence – at least 2 E